Functional UNcertainty Constrained by Law and Experiment (F U.N.C.L.E.)

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2^{nd} Order Approximation of Log Probability

Numerically find MAP estimate parameters of unknown function. For each experiment, estimate the Fisher information ($\mathcal{I}_k$) and interpret it as how the k^{th} constrains the unknown function.

Experiments $x = [x_0, \ldots, x_n]$, where x_k is data from the k^{th} experiment

Likelihood

$$p_l(x|\theta) = \prod_k p_l(x_k|\theta)$$

Prior

$$p_p(\theta)$$

A posteriori distribution

$$p(\theta|x) = \frac{p_l(x|\theta)p_p(\theta)}{\int p_l(x|\phi)p_p(\phi)d\phi}$$

MAP Maximum A posteriori Probability

$$\hat{\theta} \equiv \underset{\theta}{\operatorname{argmax}}\, p(\theta|x)$$

Taylor series

$$\begin{aligned}\log\left(p(\theta|x)\right) &= \log\left(\frac{p_l(x|\hat{\theta})p_p(\hat{\theta})}{\int p_l(x|\phi)p_p(\phi)d\phi}\right)\\ &+ \frac{1}{2}\left(\theta-\hat{\theta}\right)^T\left(\frac{d^2\log\left(p_l(x|\phi)\right)}{d\phi^2} + \frac{d^2\log\left(p_p(\phi)\right)}{d\phi^2}\right)_{\phi=\hat{\theta}}\left(\theta-\hat{\theta}\right)\\ &+ R\\ &\equiv C + \frac{1}{2}\left(\theta-\hat{\theta}\right)^T H\left(\theta-\hat{\theta}\right) + R\end{aligned}$$

Gaussian approximation

$$\theta|x \sim \mathcal{N}\left(\hat{\theta}, \Sigma = H^{-1}\right)$$

$$p(\theta|x) = \frac{1}{\sqrt{(2\pi)^{\dim}|\Sigma|}}\exp\left(-\frac{1}{2}(\theta-\hat{\theta})^{\mathrm{T}}\Sigma^{-1}(\theta-\hat{\theta})\right)$$

Fisher information $\mathcal{I}_k \equiv -\mathbb{E}_{X_k}\left[\left[\frac{\partial^2}{\partial\theta^2}\log p(X_k;\theta)\middle|\theta\right]\right]$

Bibliography

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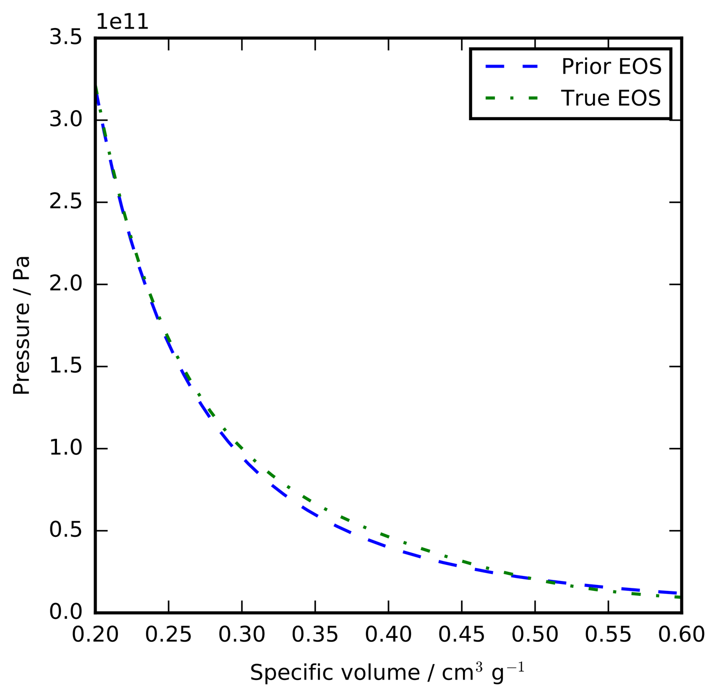
Pemberton et al. "Test Report for Equation of State Measurements of PBX-9501". LA-UR-11-04999, Los Alamos National Laboratory, Los Alamos, NM.

LA-UR-16-24568

Unknown EOS function



Pressure unknown function, $p(v)$

Spline Represent functions as cubic splines with fixed knot locations. Optimize over coefficients.

Prior Gaussian with mean coefficients, $c_f[i]$, fit to

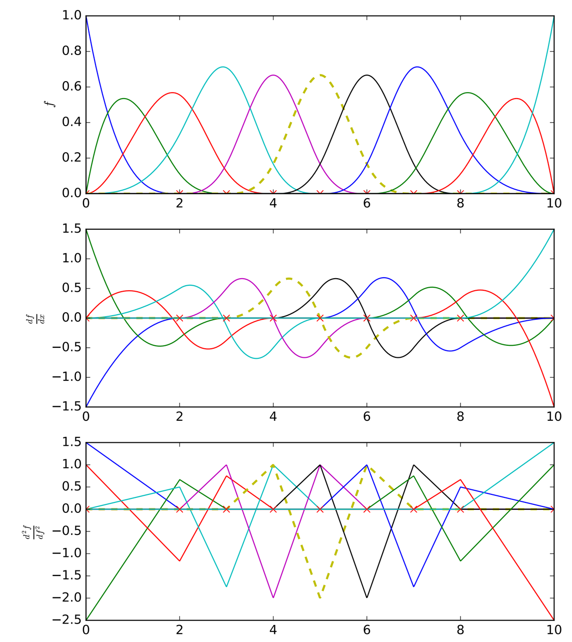
$$\tilde{f}(v) = \frac{F}{v^3}, \text{ where } F \leftrightarrow 2.56\times10^9\text{Pa at one cm}^3\text{g}^{-1}$$

and variance

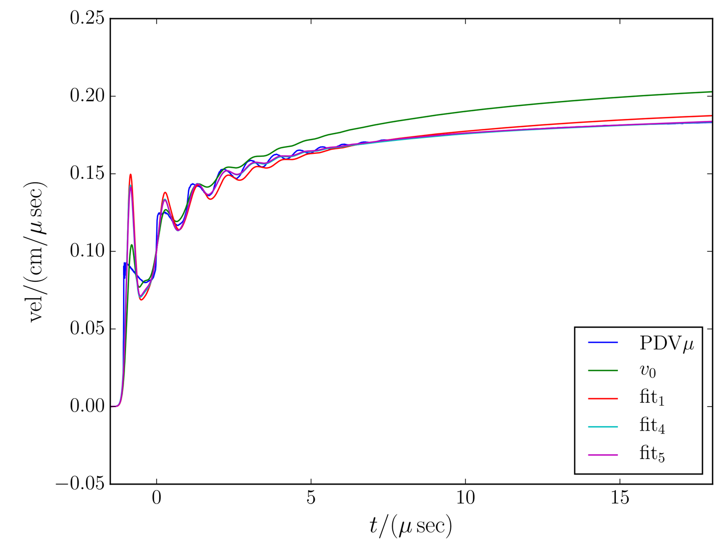
$$\sigma^2[i] = \left(0.05 \cdot c_f[i]\right)^2$$

Constraints Enforced by CVXOPT, (Not consistent with prior)

- Positive
- Monotonic
- Convex

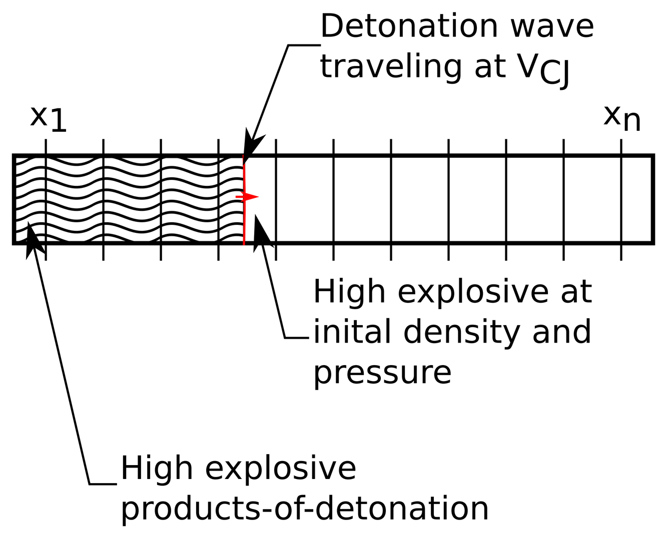


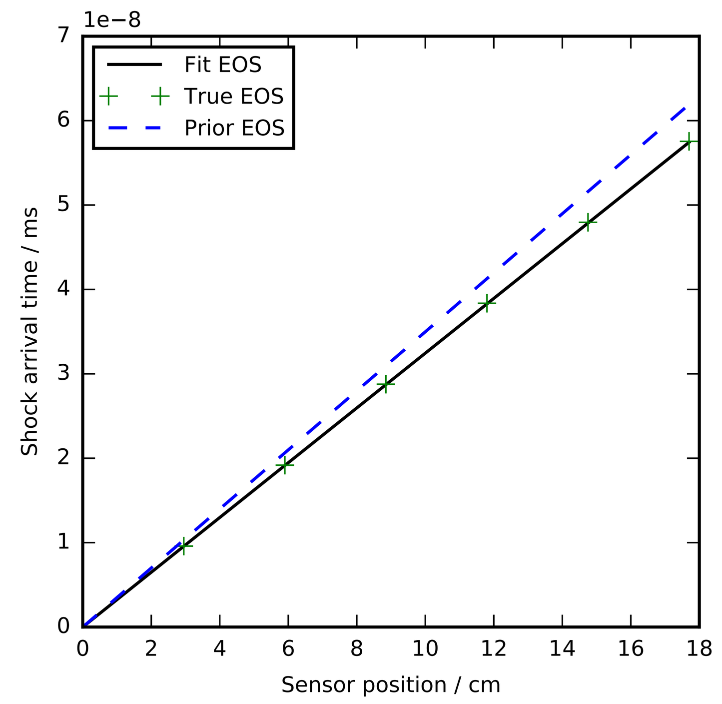
Cubic spline basis functions with first and second derivatives.

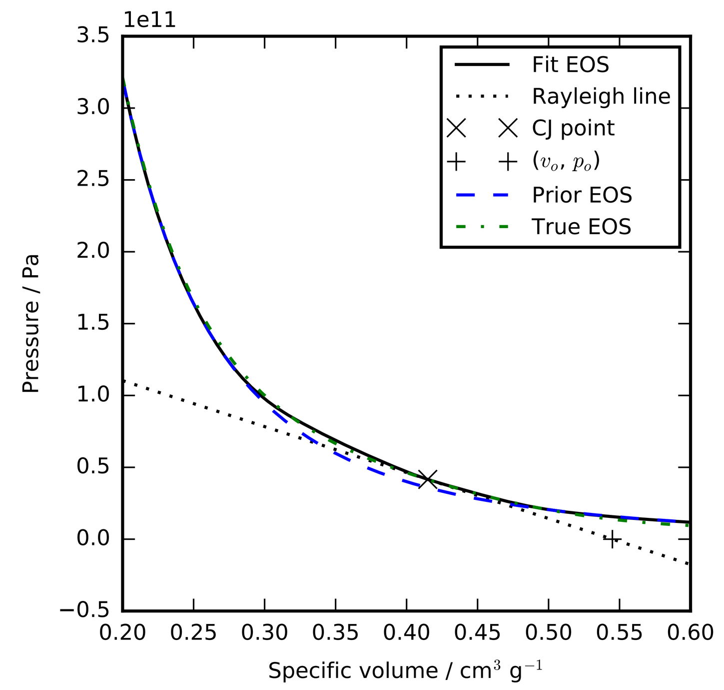


Photon Doppler velocimetry (PDV) data and sequence of fits to experiments on PBX-9501.

Rate Stick Experiment

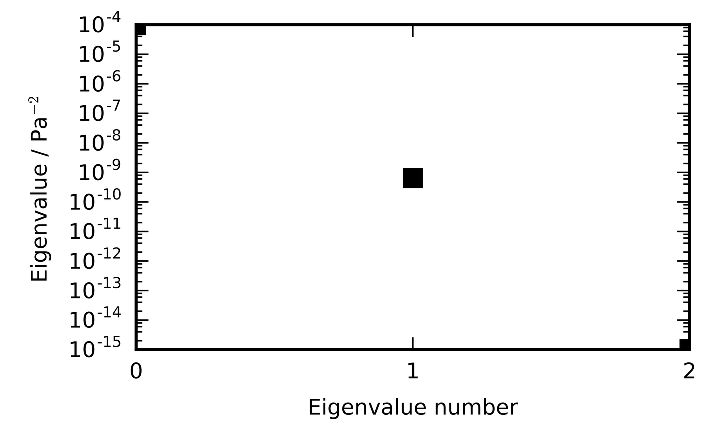


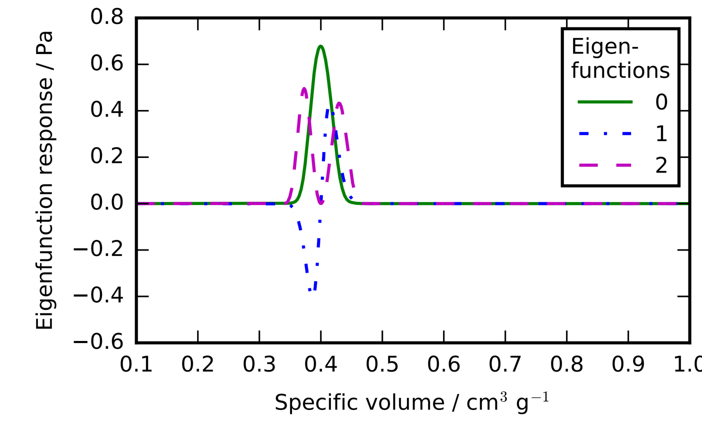




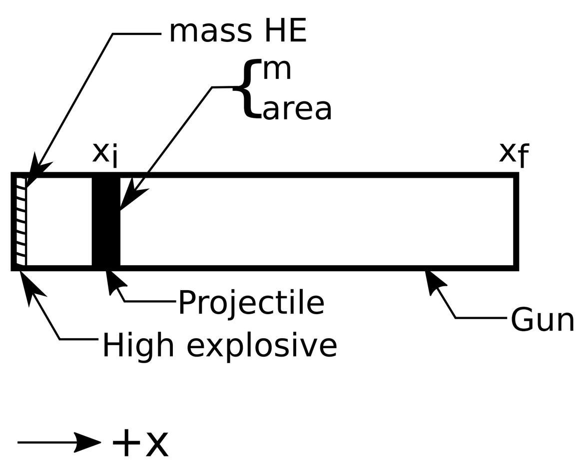
Detonation velocity from Chapman Jouguet conditions:

- *Rayleigh line* through (v_0, p_0) and tangent to EOS defines $(v_{\text{CJ}}, p_{\text{CJ}})$
- Velocity $V_{\text{CJ}} = v_0\sqrt{\frac{p_{\text{CJ}}-p_0}{v_0-v_{\text{CJ}}}}$





Gun Experiment



$$F = m \cdot a$$

$$p(v)\cdot A = m\cdot\frac{d^2}{dt^2}x$$

$$p\left(\frac{A\cdot x}{m_{H.E.}}\right)\frac{A}{m} = \frac{d^2}{dt^2}x$$

$$\begin{bmatrix}z_1\\z_2\end{bmatrix} \equiv \begin{bmatrix}x\\\frac{dx}{dt}\end{bmatrix} \equiv \begin{bmatrix}x\\V\end{bmatrix}$$

$$\frac{dz}{dt} = \begin{bmatrix}z_2\\p(A\cdot z_1)\frac{A}{m}\end{bmatrix}$$

- Sample times $\mathbf{t} = [0, \Delta_t, 2\Delta_t, \cdots, N\Delta_t]$
- scipy.integrate.odeint $\rightarrow$ $[V_0, V_1, \cdots, V_N]$
- scipy.interpolate.UnivariateSpline $\rightarrow$ $V(t)$